

Does the effective Lagrangian for low-energy QCD scale?

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QCD is not an approximately scale invariant theory. Hence a dilaton field is not expected to provide a good description of the low-energy dynamics associated with the gluon condensate. Even if such a field is introduced, it remains almost unchanged in hadronic matter at normal densities. This is because the large glueball mass together with the size of the phenomenological gluon condensate ensure that changes to that condensate are very small at such densities. Any changes in hadronic masses and decay constants in matter generated by that condensate will be much smaller than those produced directly by changes in the quark condensate. Hence masses and decay constants are not expected to display a universal scaling.

It has recently become popular to extend models intended as approximations to an effective Lagrangian for low-energy QCD by including a dilaton field [1-8]. This is done in order to make contact with the scale anomaly of QCD, as discussed by Schechter and others [9,10]. Such models have also been used to justify a universal scaling of all hadron masses and decay constants in dense matter [11].

The basic ingredient in these models is an extra scalar, isoscalar field, the dilaton [12], whose vacuum expectation value provides the only scale in the model. All dimensioned coupling constants are replaced by appropriate powers of this field multiplied by dimensionless constants. For example, in a sigma model the pion decay constant becomes a multiple of the dilaton field. The self-interaction potential for this field, denoted by χ , is taken to be of the form

$$V(\chi) = a\chi^4 + b\chi^4 \ln(\chi/\chi_0). \quad (1)$$

The first term provides a scale-invariant classical potential which on its own would lead to a vanishing vacuum expectation value for χ . It would also leave the dilaton excitations mass-

less, rather like Goldstone bosons. The second term models the quantum effects responsible for the scale anomaly. It explicitly breaks scale invariance, driving the vacuum to a nonzero value of χ and providing a mass for the dilaton excitations. The single dimensioned parameter of the model is χ_0 , which sets the scale of all other dimensioned masses and couplings. From a scaling of all dimensioned quantities, one finds that the χ field can be related to the trace of the stress-energy tensor by

$$-4b\chi^4 = T_\mu^\mu. \quad (2)$$

This trace contains all effects which break scale invariance. In QCD it is dominated by a gluonic contribution which arises from the scale anomaly [13]. The vacuum expectation value of this is given by the gluon condensate, a phenomenological value for which can be extracted from QCD sum rules [14–16].

Such a field can provide a useful description of the low-energy dynamics so long as the breaking of scale invariance is small. This is analogous to the use of PCAC and effective chiral Lagrangians to describe the interactions of pions. There one makes use of the chiral $SU(2) \times SU(2)$ symmetry of QCD which is weakly broken by the current masses of the up and down quarks. The small explicit symmetry breaking means that pions, although massive, retain much of their Goldstone boson character. In particular one can use

$$f_\pi m_\pi^2 \boldsymbol{\pi} = \partial_\mu \mathbf{A}^\mu \quad (3)$$

to define an interpolating pion field π . The matrix elements of this field are dominated by the pion pole, since all other states of the same quantum numbers lie much higher in energy. This is the basis of partial conservation of the axial current (PCAC) which can be used to relate interactions of pions with other particles to the symmetry properties of those particles. These soft pion theorems are incorporated into the effective chiral Lagrangians which form the basis of chiral perturbation theory.

Similarly, if the dilaton mass m_χ were light enough, the relation (2) could be used to define an interpolating dilaton field by analogy with the pion field of PCAC (3). This could

then be used to obtain “soft dilaton theorems” describing the consequences of approximate scale invariance. Lagrangians including this field and the potential (1) would embody this approximate symmetry.

QCD is a theory whose Lagrangian is scale invariant at the classical level (except for the current masses of the quarks) but this invariance is broken by quantum effects. This breaking is large, as can be seen from the fact that the lightest scalar glueball, which one might hope to identify with a dilaton, is estimated to lie at around 1.5 GeV [17,18]. There are many other scalar, isoscalar states in the energy range 1–2 GeV and so a single pole is most unlikely to dominate matrix elements of the stress energy tensor. Hence an interpolating dilaton field introduced in the above manner is not a useful ingredient in low-energy effective Lagrangians for QCD.

Moreover, the large mass of the scalar glueball indicates a strong restoring force against deformations of the gluon condensate. This suggests that changes to the gluon condensate are likely to be small, both in normal nuclear matter and in the exotic pionic matter discussed by Mishustin and Greiner [7]. Hence even if a dilaton field is introduced in low-energy effective Lagrangians it plays no significant role in the dynamics. This has been known since such models were first used in the context of hadron structure [1-8]: significant changes to the gluon condensate are not produced inside hadrons or normal nuclear matter if realistic values of the glueball mass and gluon condensate are used. Such small effects as do occur in models with a dilaton field should not be regarded as reliable estimates because the scale invariance is so strongly broken.

A clear demonstration of the stiffness of the gluon condensate is provided by the work of Cohen, Furnstahl and Griegel [19], which uses the trace anomaly and the Feynman-Hellmann theorem to relate the change in the gluon condensate to the energy density of hadronic matter. The trace of the stress energy tensor for QCD is given by the gluonic piece from the scale anomaly plus terms arising from the current quark masses:

$$T^\mu_\mu = -\frac{9\alpha_s}{8\pi}G^a_{\mu\nu}G^{a\mu\nu} + m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s, \quad (4)$$

where heavy quark contributions have been neglected [19]. In the vacuum this is dominated by the contribution of the gluon condensate $\langle(\alpha_s/\pi)G_{\mu\nu}^a G^{a\mu\nu}\rangle \simeq (360 \pm 20 \text{ MeV})^4$ [14–16].

In stable nuclear matter the pressure vanishes and the change in T_μ^μ is simply the energy density of the matter \mathcal{E} :

$$\langle T_\mu^\mu \rangle_\rho = \langle T_\mu^\mu \rangle_{vac} + \mathcal{E}. \quad (5)$$

Assuming that the change in the nonstrange quark condensate is given by the leading, model-independent result [20,19] and neglecting the strange quark content of the proton, the change in the gluon condensate is [19]

$$\langle(\alpha_s/\pi)G_{\mu\nu}^a G^{a\mu\nu}\rangle_\rho - \langle(\alpha_s/\pi)G_{\mu\nu}^a G^{a\mu\nu}\rangle_{vac} \simeq -\frac{8}{9}(E - \sigma_{\pi N})\rho, \quad (6)$$

where E denotes the energy per nucleon and $\sigma_{\pi N}$ the pion-nucleon sigma commutator [21]. The smallness of nuclear binding energies means that (6) is dominated by the rest masses of the nucleons and so the change in the gluon condensate is essentially proportional to the baryon density ρ .

For normal nuclear matter of density $\rho \simeq 0.17 \text{ fm}^{-3}$, this gives a change in the gluon condensate of about 150 MeV fm^{-3} . This should be compared with the vacuum gluon condensate of 2200 MeV fm^{-3} . Even allowing for a factor of two uncertainty in this condensate, its change in nuclear matter is at most a 15% effect. The fourth root of the condensate, which corresponds to the change in the dilaton field or the change of scale, is altered by no more than 4%. For the pionic droplets studied in [7] the energy density is even smaller, about 20 MeV fm^{-3} and so the dilaton field is barely changed.

There are only two ways to get large changes in the gluon condensate at normal densities relative to its vacuum value. One is to take a value for the vacuum condensate very much smaller than that deduced from QCD sum rules. That would mean rejecting the rather well tested applications of those sum rules to charmonium [14–16]. The other is to use a χ field with a very light mass so that the vacuum is soft in this channel and the response is large. That would mean returning to the light dilaton idea, even though no such particle is observed and both lattice calculations and hadron spectroscopy suggest a scalar glueball

mass of about 1.5 GeV [17,18]. It would also be inconsistent with observed nuclear binding energies, since Eq. (6) provides a connection between these and the change in the gluon condensate. Neither of these choices seems acceptable.

In summary: QCD is not an approximately scale invariant theory and hence a dilaton field does not provide a good description of the low-energy dynamics associated with the gluon condensate (unlike the pion in the context of chiral dynamics). Moreover, the large glueball mass together with the size of the phenomenological gluon condensate mean that changes to that condensate are very small in hadronic matter at normal densities.

A corollary to this is that hadron masses and decay constants do not scale in matter as suggested by Brown and Rho [11]. Any changes in these quantities are likely to be driven directly by the reduction of the quark condensate. The model-independent result for the linear dependence of the quark condensate on density [20,19] shows that large changes in that condensate can occur independently of any change in the gluon condensate. The fact that different condensates behave very differently at finite density should not be too surprising: there are many possible energy scales in matter which can be constructed from those condensates and the density.

Scaling could only be recovered if one were to use a model where the lightest scalar meson had a much larger mass than the dilaton, as noted by Kusaka and Weise [5,22].¹ The quark condensate would then be very stiff and would not respond directly to the scalar density of quarks in matter. Any changes to it could only arise from changes to the gluon condensate, and hence would be very small for the reasons described above. The size of the

¹The scaling hypothesis leads to hadron masses which vary as the cube root of the quark condensate. Such a relationship has also been found in a version of the NJL model without taking a very large mass for the scalar meson [4]. However in that model the relationship between the masses and the quark condensate is not a consequence of scaling but instead arises from the artificial choice of a model involving four-body rather than two-body forces between the quarks.

πN sigma commutator and its associated form factor [21] indicate that the quark condensate is in fact significantly deformed in the presence of valence quarks. This can occur even if the “elementary” scalar meson is heavy because of its strong mixing with the two pion channel [23].

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- [1] Gomm H, Jain P, Johnson R and Schechter J 1986 *Phys. Rev.* **D33** 3476
 Meissner U-G and Kaiser N 1987 *Phys. Rev.* **D35** 2859
 Meissner U-G, Johnson R, Park N and Schechter J 1988 *Phys. Rev.* **D37** 1285
 - [2] Jain P, Johnson R and Schechter J 1988 *Phys. Rev.* **D38** 1571
 - [3] Ellis J, Kapusta J I and Olive K A 1991 *Nucl. Phys.* **B348** 345; *Phys. Lett.* **B273** 123
 - [4] Ripka G and Jaminon M 1992 *Ann. Phys., NY* **218** 51
 Jaminon M and Ripka G 1993 *Nucl. Phys.* **A564** 551
 - [5] Kusaka K and Weise W 1992 *Z. Phys.* **A343** 229; *Phys. Lett.* **B288** 6
 - [6] Mishustin I, Bondorf J and Rho M 1993 *Nucl. Phys.* **A555** 215
 - [7] Mishustin I N and Greiner W 1993 *J. Phys. G: Nucl. Part. Phys.* **19** L101
 - [8] Waizdych T and Wambach J 1994 Private communication

- [9] Schechter J 1980 *Phys. Rev.* **D21** 3393
- Migdal A A and Shifman M A 1982 *Phys. Lett.* **B114** 445
- [10] Gomm H, Jain P, Johnson R and Schechter J 1986 *Phys. Rev.* **D33** 801, and references therein
- [11] Brown G E and Rho M 1991 *Phys. Rev. Lett.* **66** 2720
- [12] Ellis J 1970 *Nucl. Phys.* **B22** 478
- [13] Collins J C, Duncan A and Joglekar S D 1977 *Phys. Rev.* **D16** 438
- Nielsen N K 1977 *Nucl. Phys.* **B210** 212
- [14] Shifman M A, Vainshtein A I and Zakharov V I 1979 *Nucl. Phys.* **B147** 385, 448
- [15] Reinders L J, Rubinstein H and Yazaki S 1985 *Phys. Reports* **127** 1
- [16] Narison S 1989 *QCD spectral sum rules* (Singapore: World Scientific)
- [17] Bali G S, Schilling K, Hulsebos A, Irving A C, Michael C and Stephenson P W 1993 *Phys. Lett.* **B309** 378
- [18] Morgan D 1993 *RAL report* RAL-93-078
- [19] Cohen T D, Furnstahl R J and Griegel D K 1992 *Phys. Rev.* **C45** 1881
- [20] Drukarev E G and Levin E M 1990 *Nucl. Phys.* **A511** 679; **A516** 715(E)
- [21] Gasser J, Leutwyler H and Sainio M E 1991 *Phys. Lett.* **B253** 252, 260
- [22] Kusaka K and Weise W 1993 *University of Regensburg preprint* TPR-93-40
- [23] Birse M C 1994 *Phys. Rev.* **C49** (in press)